

Section 5.3 – I.C.E – Solving Trig Equations

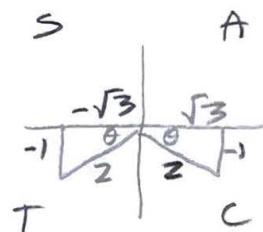
Solve the following trig equations and give all possible solutions in the interval $[0, 2\pi]$.

1. $2\sin\theta + 1 = 0$

$$2\sin\theta = -1$$

$$\sin\theta = -\frac{1}{2}$$

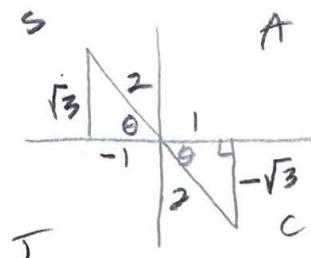
$$\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$$



2. $\tan\theta + \sqrt{3} = 0$

$$\tan\theta = -\sqrt{3}$$

$$\theta = \frac{2\pi}{3}, \frac{5\pi}{3}$$



3. $2\sin^2\theta + 3\sin\theta + 1 = 0$

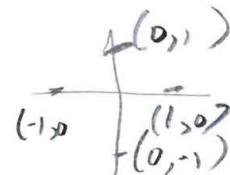
$$(2\sin\theta + 1)(\sin\theta + 1) = 0$$

$$\sin\theta = -\frac{1}{2}$$

$$\sin\theta = -1$$

$$\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$$

$$\theta = \frac{3\pi}{2}$$

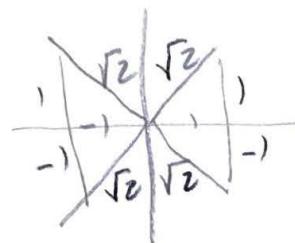


4. $2\sin^2\theta = 1$

$$\sin^2\theta = \frac{1}{2}$$

$$\sin\theta = \pm\frac{1}{\sqrt{2}}$$

$$\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$



5. $2\cos\theta\sin\theta - \cos\theta = 0$

$$\cos\theta(2\sin\theta - 1) = 0$$

$$\cos\theta = 0$$

$$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$\sin\theta = \frac{1}{2}$$

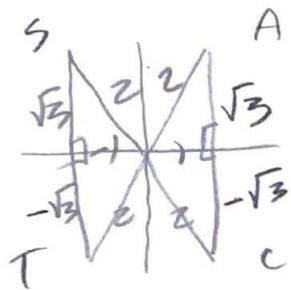
$$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$$

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6. $\sin^2 \theta = 3\cos^2 \theta$

$$\begin{aligned} 1 - \cos^2 \theta &= 3\cos^2 \theta \\ 1 &= 4\cos^2 \theta \\ \frac{1}{4} &= \cos^2 \theta \\ \cos \theta &= \pm \frac{1}{2} \end{aligned}$$

$$\begin{aligned} \theta &= \frac{\pi}{3}, \frac{2\pi}{3}, \\ &\frac{4\pi}{3}, \frac{5\pi}{3} \end{aligned}$$



7. $\sec \theta \csc \theta = 2 \csc \theta$

$$\begin{aligned} \sec \theta \csc \theta - 2 \csc \theta &= 0 \\ \csc \theta (\sec \theta - 2) &= 0 \\ \csc \theta &= 0 \quad \sec \theta = 2 \\ \frac{1}{\sin \theta} &= 0 \\ \sin \theta &= \text{undefined} \end{aligned}$$

$$\begin{aligned} \cos \theta &= \frac{1}{2} \\ \theta &= \frac{\pi}{3}, \frac{5\pi}{3} \end{aligned}$$

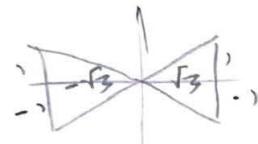
8. $\sec^2 x - 1 = 0$

$$\begin{aligned} \sec^2 x &= 1 \\ \sec x &= \pm 1 \quad x = 0, \pi \\ \frac{1}{\cos x} &= \pm 1 \end{aligned}$$

9. $(3\tan^2 x - 1)(\tan^2 x - 3) = 0$

$$\begin{aligned} \tan^2 x &= \frac{1}{3} \quad \tan^2 x = 3 \\ \tan x &= \pm \frac{1}{\sqrt{3}} \quad \tan x = \pm \sqrt{3} \end{aligned}$$

$$x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}, \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$



10. $2\sin^2 x = 2 + \cos x$

$$2(1 - \cos^2 x) - \cos x - 2 = 0$$

$$2 - 2\cos^2 x - \cos x - 2 = 0$$

$$2\cos^2 x + \cos x = 0$$

$$\cos x(2\cos x + 1) = 0$$

$$\cos x = 0$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$\cos x = -\frac{1}{2}$$

$$x = \frac{2\pi}{3}, \frac{4\pi}{3}$$